Multiple View Reconstruction of a Quadric of Revolution from its Occluding Contours

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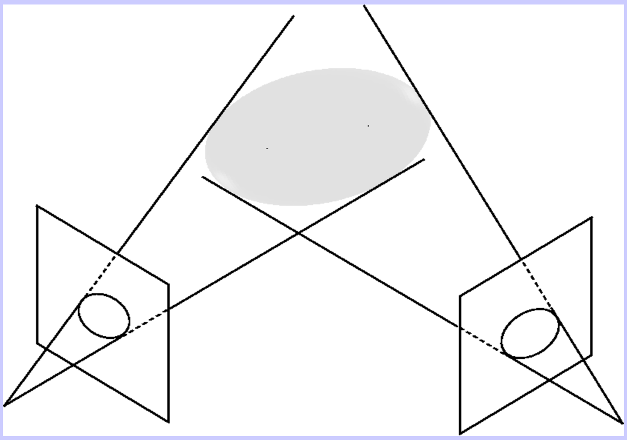
Problem : Find a linear triangulation scheme to reconstruct a *quadric of revolution* (QoR) from its occluding contours in at least 2 views.



A quadric is the envelope of planes p satisfying :

$$p^T Q^* p = 0$$

with Q* a 4x4 symmetric matrix.

Key idea

We write a QoR as :

$$Q^* = X^* - x_0 Q^*_\infty$$

with $X^* = FG^T + GF^T$ is the rank-2 quadric dual to the principal focus-pair (F,G),
Q^*_∞ is the matrix of the Dual Absolute Quadric.

The image of the QoR writes :

$$C^* = PQ^*P^T \triangleq Y^* - \beta\omega^*$$

with $Y^* = fg^T + gf^T$ is the rank-2 conic dual to the images f,g of principal foci F,G,
$\omega^* = KK^T$ is the dual image of the absolute conic.

Key result

Proposition : If the occluding contour C of a « prolate » QoR is given in a calibrated view, then the images (f , g) of the real principal foci of the QoR can be set-wise recovered from C* and ω^* i.e., the degenerate dual conic can be uniquely recovered.

Triangulation equations

(a) Let l be the line tangent to the image of the quadric :

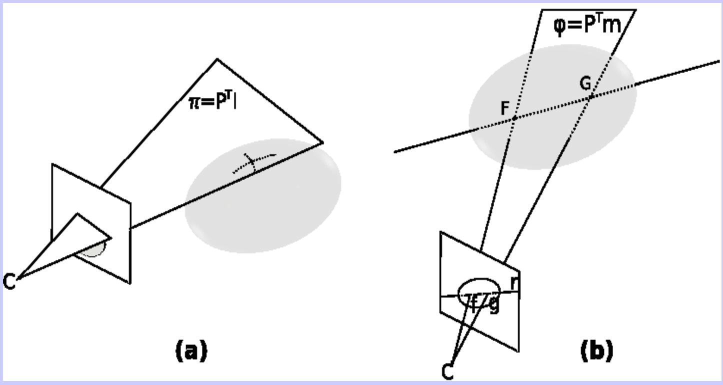
$$\pi^T X^* \pi - x_0 (\pi^T Q^*_\infty \pi) = 0$$

where $\pi = P^T l$.

(b) Let r be the line passing through f and g :

$$X^* \varphi = 0$$

where $\varphi = P^T r$.



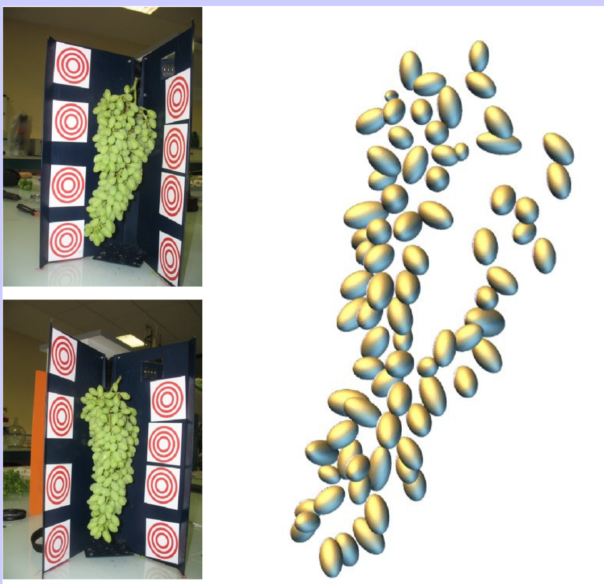
For each view i from 1 to N :

$$\varphi_i = P_i^T r_i \quad \text{and} \quad \pi_i^j = P_i^T l_i^j \ (j=1,2)$$

Solution

$$\min_{X^*, x_0} \sum_{i=1}^{N} \{ \|X^* \varphi_i\|^2 + \sum_{j=1}^{2} \| \pi_i^{jT} X^* \pi_i^j - x_0 (\pi_i^{jT} Q^*_\infty \pi_i^j) \|^2 \}$$

Application with a bunch of grapes :



Conclusion

We described a multiple view algorithm that unambiguously reconstructs so-called prolate quadrics of revolution, given at least two finite projective cameras. This method has been used to estimate the volume of a bunch of grapes. This is a preliminary work of my upcoming PhD in september 2009.

Figeac, le 16 juin 2009